

Semiclassical long throats of the wormholes¹

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The vacuum polarization of the quantized scalar field with non-conformal coupling ξ on the long throat of a wormhole background is calculated. It is shown that the stress-energy tensor of vacuum fluctuations in considered spacetime is determined by the local geometry of spacetime only. The self-consistent solutions of the semiclassical Einstein field equations describing a long throat of a traversable wormhole are obtained.

1 Introduction

By definition a wormhole is a bridge connecting two asymptotically flat regions. Usually such construction is a classical object and should satisfy to Einstein equations. The topology of the 4D wormholes is the topology of direct product of the Minkowski plane and a unit sphere. Static traversable wormholes could be threaded by "exotic matter" that violates certain energy conditions at least at the throat [1]. As an example of such a matter one can consider the vacuum of quantized fields. This approach gives the possibility to consider the wormhole metric as the self-consistent solution of the semiclassical theory of gravity. In the realm of this theory the vacuum fluctuations of the quantized fields are the source of spacetime curvature³

$$G_{\nu}^{\mu} = 8\pi \langle T_{\nu}^{\mu} \rangle_{ren}. \quad (1)$$

The main difficulty in the theory of semiclassical gravity is that the effects of the quantized gravitational field are ignored. The popular solution of this problem is to justify ignoring the gravitational contribution by working in the limit of a large number of fields, in which the gravitational contribution is negligible. Another problem is that the vacuum polarization effects are determined by the topological and geometrical properties of spacetime as a whole or by the choice of quantum state in which the expectation values are taken. It means that calculating of the functional dependence of $\langle T_{\nu}^{\mu} \rangle_{ren}$ on

the metric tensor in an arbitrary spacetime presents formidable difficulty. Only in some spacetimes with high degrees of symmetry for the conformally invariant fields $\langle T_{\mu\nu} \rangle_{ren}$ can be computed and equations (1) can be solved exactly [3]. Let us stress that the single parameter of length dimensionality in such a problem is the Planck length l_{Pl} . This implies that the characteristic scale l of the spacetime curvature (which correspond to the solution of equations (1)) can differ from l_{Pl} only if there is a large dimensionless parameter. As an example of such a parameter one can consider a number of fields the polarization of which is a source of spacetime curvature⁴. In some cases $\langle T_{\mu\nu} \rangle_{ren}$ is determined by the local properties of a spacetime and it is possible to calculate the functional dependence of the renormalized expression for the vacuum expectation value of the stress-energy tensor operator of the quantized fields on the metric tensor approximately. One of the most widely known examples of such a situation is the case of a very massive field. In this case $\langle T_{\nu}^{\mu} \rangle_{ren}$ can be expanded in terms of powers of the small parameter

$$\frac{1}{ml} \ll 1, \quad (2)$$

where m is the mass of the quantized field and l is the characteristic scale of the spacetime curvature [4, 5]. Using the first nonvanishing term of this expansion for minimally or conformally coupled scalar field Taylor, Hiscock and Anderson [6] have showed that the equations (1) have no wormhole solution for some class of static spherically

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³throughout we use units such that $c = \hbar = G = 1$.

⁴Here and below it is assumed, of course, that the characteristic scale of change of the background gravitational field is sufficiently greater than l_{Pl} so that the very notion of a classical spacetime still has some meaning.

symmetric spacetimes. Let us stress that in this case the existence of an additional parameter of the length dimensionality $1/m$ does not increase the characteristic scale of the spacetime curvature which is described by the solution of equations (1)⁵.

It is necessary to note that different quantum field theory constraints on the parameters of traversable wormholes are known [7]. As a rule these constraints were derived for a massless scalar field.

The purpose of this paper is to examine whether the vacuum fluctuations of quantized fields in the one-loop approximation in Einstein's theory can create traversable Lorentzian wormholes. One of the possible ways to solve this problem is to use the analytical approximation for the expectation value of the stress-energy tensor operator of the quantized matter fields in curved spacetimes [5, 8]. This approach was realized in the works [9]. The problem of such an approach is the uncertainty of the applicability limits of such approximations. As it was noted by Khatsymovsky [10] in the spacetime that is a direct product of the Minkowski plane and a two-dimensional sphere of a fixed radius (the topologies of this spacetime and wormhole spacetime coincide) these approximations are not applicable. Another way to obtain the wormhole solution of equations (1) is to use the model of short-throat flat-space wormhole [11] (see also [12]). This model represents two identical copies of Minkowski spacetime with spherical regions excised from each copy and so that points of these regions are identical. One can consider this model as the first approximation of real situation if there is a small parameter L/r , where L is the length of the throat and r is the radius of the throat. At the present time only the full vacuum energy of quantized scalar field have been calculated in this spacetime. Nevertheless it gives a possibility to make some evaluations of the radius of a wormhole throat. A completely opposite model is the model of a long-throat wormhole. Local approximations for $\langle T_\nu^\mu \rangle$ in the throat of static spherically symmetric long-throat wormhole were obtained in [13] for massless fields of spin 1 and 1/2. The results of these works were obtained by the WKB method and the small parameter of these approximations is the ratio of the throat radius to the length of

the throat. This result gives the possibility to reply to the question: can the throat of a long-throat wormhole be created by the vacuum fluctuations of quantum fields?

In this paper, the results of the work [14] are used to evaluate the local approximation of the renormalized expression for the vacuum expectation value of the stress-energy tensor operator of the quantized scalar field in the throat of static spherically symmetric long-throat wormhole spacetime (Sec.2). In Sec.3 the solution of semiclassical Einstein field equations describing the long throat of a traversable Lorentzian wormhole are given. Sec.4 summarizes the contents.

2 Stress-energy of a quantized scalar field in spacetime of static spherically symmetric long throat wormhole

The line element of a static spherically symmetric wormhole spacetime can be written as

$$ds^2 = -f dt^2 + d\rho^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (3)$$

where $-\infty < \rho < \infty$ and $f = f(\rho)$, $r = r(\rho)$.

The vacuum polarization effects of a quantized field can be described approximately [13, 14] in the region of variation of ρ where the functions $f(\rho), r(\rho)$ change slowly. One can consider such a region as a long throat of a wormhole. It is necessary to note that the notion of throat's length is not well-defined. To discuss this notion let us consider the model of wormhole

$$ds^2 = -dt^2 + d\rho^2 + [\rho \tanh(\rho/\lambda) + r_0]^2 (d\theta^2 + \sin^2\theta d\phi^2). \quad (4)$$

In this model the parameter r_0 describes the radius of the throat. In the region $|\rho| \gg \lambda$ the spacetime is asymptotically flat. A curved region $|\rho| \lesssim \lambda$ can be called the throat of the wormhole and λ characterizes the length of this throat (see, also, [15]). In a more general case the length of the throat can be defined as a scale characterizing the variation of the metric function $r^2(\rho)$ at the throat provided that this scale is much more than the radius of the throat. Additionally one should assume that $f(\rho)$ is changed slowly.

The renormalized expressions for the vacuum expectation value of the stress-energy tensor operator of the quantized scalar field in the region of

⁵The characteristic scale of the components G_ν^μ on the left-hand side of equations (1) is $1/l^2$, on the right-hand side - $l_{Pl}^2/(m^2 l^6)$.

variation of ρ where the functions $f(\rho)$ and $r(\rho)$ change slowly can be expanded (see [14]) in terms of powers of the small parameter

$$L_*(\rho)/L(\rho) \ll 1, \quad (5)$$

where

$$L_*(\rho) = \left[m^2 + \frac{2\xi}{r^2} \right]^{-1/2}, \quad (6)$$

m is the mass of the scalar field, ξ is its coupling to the scalar curvature and $L(\rho)$ is a scale of variation of the metric functions:

$$\frac{1}{L(\rho)} = \max \left\{ \left| \frac{r'}{r} \right|, \left| \frac{f'}{f} \right|, \sqrt{|\xi|} \left| \frac{r'}{r} \right|, \sqrt{|\xi|} \left| \frac{f'}{f} \right|, \left| \frac{r''}{r} \right|^{1/2}, \left| \frac{f''}{f} \right|^{1/2}, \sqrt{|\xi|} \left| \frac{r''}{r} \right|^{1/2}, \dots \right\}. \quad (7)$$

The zeroth-order terms (with respect to a small parameter L_*/L) of the renormalized expression for the vacuum expectation value of the stress-energy tensor operator of the quantized scalar field in such a spacetime is given by (see [14])

$$\begin{aligned} \langle T_t^t \rangle_{ren} = \langle T_\rho^\rho \rangle_{ren} = & \frac{1}{4\pi^2 r^4} \left\{ \frac{m^2 r^2}{8} \left(\xi - \frac{1}{8} \right) \right. \\ & + \frac{79}{7680} - \frac{11}{96} \xi + \frac{3}{8} \xi^2 + \left[-\frac{m^4 r^4}{8} + \frac{m^2 r^2}{2} \left(\frac{1}{6} - \xi \right) \right. \\ & - \frac{1}{60} + \frac{1}{6} \xi - \frac{1}{2} \xi^2 \left. \right] \ln \sqrt{\frac{\mu^2}{m_{DS}^2 r^2}} + \left[2m^2 r^2 \left(\xi - \frac{1}{8} \right) \right. \\ & \left. \left. + \frac{m^4 r^4}{2} + 2 \left(\xi - \frac{1}{8} \right)^2 \right] [I_1(\mu) - I_2(\mu)] \right\}, \quad (8) \end{aligned}$$

$$\begin{aligned} \langle T_\theta^\theta \rangle_{ren} = \langle T_\varphi^\varphi \rangle_{ren} = & \frac{1}{4\pi^2 r^4} \left\{ \frac{m^2 r^2}{8} \left(\xi - \frac{1}{8} \right) \right. \\ & + \left[-\frac{m^4 r^4}{8} - \frac{1}{8} \left(\xi - \frac{1}{8} \right)^2 + \frac{1}{60} - \frac{1}{6} \xi \right. \\ & + \frac{1}{2} \xi^2 \left. \right] \ln \sqrt{\frac{\mu^2}{m_{DS}^2 r^2}} + \left[m^2 r^2 \left(\frac{1}{8} - \xi \right) \right. \\ & - 2 \left(\xi - \frac{1}{8} \right)^2 \left. \right] I_1(\mu) + \left[\frac{m^4 r^4}{2} \right. \\ & \left. + 2m^2 r^2 \left(\xi - \frac{1}{8} \right) + 2 \left(\xi - \frac{1}{8} \right)^2 \right] I_2(\mu) \right\}, \quad (9) \end{aligned}$$

$$\langle T_\nu^\mu \rangle_{ren} = 0, \quad \mu \neq \nu, \quad (10)$$

where

$$\mu^2 = m^2 r^2 + 2\xi - 1/4 > 0, \quad (11)$$

$$\begin{aligned} I_1(\mu) &= \int_0^\infty \frac{x \ln |1 - x^2|}{1 + e^{2\pi|\mu|x}} dx, \\ I_2(\mu) &= \int_0^\infty \frac{x^3 \ln |1 - x^2|}{1 + e^{2\pi|\mu|x}} dx, \end{aligned} \quad (12)$$

m_{DS} is equal to the mass m of the field for a massive scalar field. For a massless scalar field it is an arbitrary parameter due to the infrared cutoff in renormalization counterterms for $\langle T_\nu^\mu \rangle$. A particular choice of the value of m_{DS} corresponds to a finite renormalization of the coefficients of terms in the gravitational Lagrangian and must be fixed by experiment or observation. Note that the renormalized expectation values of the stress-energy tensor components (8-10) are exact if $f(\rho) = const$ and $r(\rho) = const$. Note also that $\langle T_\nu^\mu \rangle_{ren}$ is conserved

$$\langle T_\nu^\mu \rangle_{ren; \mu} = 0 \quad (13)$$

and, for the conformally invariant field, has a trace equal to the known trace anomaly [16]

$$\begin{aligned} \langle T_\mu^\mu \rangle_{ren} = & \frac{-1}{2880\pi^2} \left[C_{\alpha\beta\gamma\delta} C^{\alpha\beta\gamma\delta} + R_{\alpha\beta} R^{\alpha\beta} \right. \\ & \left. - \frac{1}{3} R^2 - \square R \right] = -\frac{1}{4\pi^2 r^4} \frac{1}{360}. \end{aligned} \quad (14)$$

The expressions (8) and (9) may be simplified in the cases of massless field. In particular, if

$$m = 0, \xi = 1/6, L^2 \gg \frac{r^2}{2\xi}, \quad (15)$$

then

$$\begin{aligned} \langle T_t^t \rangle_{ren} = \langle T_\rho^\rho \rangle_{ren} \simeq & \frac{1}{4\pi^2 r^4} \left[0.00310 \right. \\ & \left. + \frac{1}{720} \ln(m_{DS}^2 r^2) \right], \end{aligned} \quad (16)$$

$$\begin{aligned} \langle T_\theta^\theta \rangle_{ren} = \langle T_\varphi^\varphi \rangle_{ren} \simeq & \frac{1}{4\pi^2 r^4} \left[-0.00171 \right. \\ & \left. - \frac{1}{720} \ln(m_{DS}^2 r^2) \right]. \end{aligned} \quad (17)$$

In the case of a very massive field

$$m^2 r^2 \gg |2\xi|, \quad \mu^2 \gg 1, \quad L^2 \gg \frac{r^2}{2\xi} \quad (18)$$

the expressions for $I_n(\mu)$ and $\ln(\sqrt{\mu^2/m_{DS}^2 r^2})$ can be expanded in terms of powers of $(2\xi - 1/4)/(m^2 r^2)$.

As a result the expressions (8-10) can be written in the form

$$\langle T_\nu^\mu \rangle_{ren} = \frac{1}{4\pi^2 r^4} \left[\frac{1}{m^2 r^2} \left(\frac{\xi^3}{6} - \frac{\xi^2}{12} + \frac{\xi}{60} - \frac{1}{630} \right) + O \left(\frac{(2\xi - 1/4)^2}{m^4 r^8} \right) \right] \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}. \quad (19)$$

These expressions can be obtained directly from the correspondent terms of the DeWitt-Schwinger approximation if we assume that $f = const$ and $r^2 = const$.

3 Solutions

As it was mentioned above the right hand side of equations (1) may be expanded in terms of powers of the small parameter L_\star/L in the region of variation of ρ where the metric functions $f(\rho)$ and $r(\rho)$ change slowly. This implies that these equations may be solved iteratively. And to obtain the zeroth-order solution of equations (1) one must omit the terms of order $1/L^2$ in left hand side of these equations the same way as the corresponded terms in right hand side was omitted (in expressions for $\langle T_\nu^\mu \rangle_{ren}$ (8,9))

$$\begin{aligned} G_t^t &= -\frac{1}{r^2} - \frac{(r^2)'^2}{4r^4} + \frac{(r^2)''}{r^2} = -\frac{1}{r^2} + O(1/L^2), \\ G_\rho^\rho &= -\frac{1}{r^2} + \frac{(r^2)'^2}{4r^4} + \frac{f'(r^2)'}{2fr^2} = -\frac{1}{r^2} + O(1/L^2), \\ G_\theta^\theta &= G_\varphi^\varphi = \frac{(r^2)''}{2r^2} + \frac{f''}{2f} + \frac{f'(r^2)'}{4fr^2} - \frac{(r^2)'^2}{4r^4} \\ &\quad - \frac{f''}{4f^2} = O(1/L^2). \end{aligned} \quad (20)$$

The zeroth-order solution is valid in the region of variation of ρ where $L_\star(\rho)/L(\rho) \ll 1$. As it was mentioned above one can consider such a region as a long throat of a wormhole.

The principal problem of the solution of equations (1) in considered case is that we have two equations on only one unknown variable r^2 . This problem can be overcome by the insertion into consideration the classical electrostatic field. The stress-energy tensor of such a field created by the charge Q in coordinates (3) is

$$T_\nu^\mu = \frac{Q^2}{8\pi r^4} \text{diag}(-1, -1, 1, 1). \quad (21)$$

Thus the parameter Q plays a role of an additional variable.

Now let us consider the case in which the right hand side of equations (1) is determined by this electrostatic field and two quantized scalar fields: conformally invariant field ($\xi_1 = 1/6, m_1 = 0$) and very massive field ($\xi_2 \equiv \xi$ - arbitrary constant, $m_2 \equiv m, \mu_2^2 = m^2 r^2 + 2\xi - 1/4 \gg 1, m^2 r^2 \gg |2\xi - 1/4|$). For this case the Einstein's equations have the form

$$-\frac{1}{8\pi r^2} \simeq \frac{1}{4\pi^2 r^4} \left[0.00310 + \frac{1}{720} \ln(m_{ds}^2 r^2) + \frac{1}{m^2 r^2} \left(-\frac{\xi^3}{6} + \frac{\xi^2}{12} - \frac{\xi}{60} + \frac{1}{630} \right) \right] - \frac{Q^2}{8\pi r^4}, \quad (22)$$

$$0 \simeq \frac{1}{4\pi^2 r^4} \left[-0.00171 - \frac{1}{720} \ln(m_{ds}^2 r^2) + \frac{1}{m^2 r^2} \left(\frac{\xi^3}{3} - \frac{\xi^2}{6} + \frac{\xi}{30} - \frac{1}{315} \right) \right] + \frac{Q^2}{8\pi r^4}. \quad (23)$$

The consequence of these equations is

$$r^4 + \frac{r^2}{360\pi} + \frac{1}{\pi m^2} \left(\frac{\xi^3}{3} - \frac{\xi^2}{6} + \frac{\xi}{30} - \frac{1}{315} \right) \simeq 0. \quad (24)$$

The solution of this equation which satisfies the condition

$$r^2 \gg 1/(720\pi) \quad (25)$$

is

$$r^2 \simeq \sqrt{-\frac{1}{\pi m^2} \left(\frac{\xi^3}{3} - \frac{\xi^2}{6} + \frac{\xi}{30} - \frac{1}{315} \right)}. \quad (26)$$

The second independent equation of system (22,23) imposes a constraint on Q^2

$$Q^2 \simeq \frac{2}{\pi} \left\{ 0.00171 + \frac{1}{720} \ln \left[\frac{m_{ds}^2}{m\sqrt{\pi}} \left(-\frac{\xi^3}{3} + \frac{\xi^2}{6} - \frac{\xi}{30} + \frac{1}{315} \right)^{1/2} \right] + \frac{\sqrt{\pi}}{m} \left(-\frac{\xi^3}{3} + \frac{\xi^2}{6} - \frac{\xi}{30} + \frac{1}{315} \right)^{1/2} \right\}. \quad (27)$$

If we take into account (25) and $m^2 r^2 \gg |2\xi - 1/4|$, the conditions of validity of this solution can be written as follows

$$\begin{aligned} -\frac{\pi(2\xi - 1/4)^2}{\left(\frac{\xi^3}{3} - \frac{\xi^2}{6} + \frac{\xi}{30} - \frac{1}{315} \right)} &\ll m^2 \ll \\ -518400\pi \left(\frac{\xi^3}{3} - \frac{\xi^2}{6} + \frac{\xi}{30} - \frac{1}{315} \right). \end{aligned} \quad (28)$$

It is necessary to remember that the region of validity of the semiclassical theory of gravity is determined by the condition $r \gg 1$. All these conditions are valid for $\xi < 0$, $|\xi| \gg 1$. A particular solution of system (22,23) is

$$\xi = -10^4, \quad m^2 = 10^3, \quad r \simeq 101.49. \quad (29)$$

Let us note that the stress-energy of the fields considered here have the needed "exotic" properties (in the sense of Morris and Thorne [1]) to support the long throat of a wormhole:

$$\begin{aligned} p_r = -\epsilon = & \frac{1}{4\pi^2 r^4} \left[0.00310 + \frac{1}{720} \ln(m_{DS}^2 r^2) \right. \\ & + \frac{1}{m^2 r^2} \left(-\frac{\xi^3}{6} + \frac{\xi^2}{12} - \frac{\xi}{60} + \frac{1}{630} \right) \Big] \\ & - \frac{Q^2}{8\pi r^4} < 0, \end{aligned} \quad (30)$$

where p_r is the radial pressure, ϵ is the energy density and r, Q are determined by expressions (26,27).

4 Conclusions

In this paper we succeed in finding the solution in semiclassical theory of gravity which describes the long throat of the wormhole. Such objects are created by the electrostatic field and the vacuum fluctuations of quantized scalar fields. The stress-energy tensor of these fluctuations in considered spacetime is determined by the local geometry of spacetime only. The geometry of spacetime far from the throat is not described by the obtained zeroth-order (with respect to the small parameter L_*/L) solution. The obvious defect of such solutions is as follows: the value of $|\xi|$ which corresponds to the large (with respect to the Planck length) value of throat radius is also large (with respect to 1). The latter seems unlikely. Nevertheless in the "large N " case, in which the number of matter fields is large, the radius of throat r is proportional to \sqrt{N} ($N^{1/4}$ for the very massive fields) and the value of r which satisfy to the condition $r \gg 1$ (i.e. much more than the Planck length) can be obtained for the value of $|\xi|$ lesser than one considered above.

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